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Luca Lambertini
Andrea Mantovani

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Feedback equilibria in a dynamic renewable resource oligopoly: pre-emption, voracity and exhaustion[1]

Luca Lambertini[a,b] and Andrea Mantovani[a,c]

a Department of Economics, University of Bologna, Italy

b ENCORE, University of Amsterdam, The Netherlands

c Barcelona Institute of Economics (IEB)

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Abstract

We extend Fujiwara's (2008) model to describe a differential oligopoly game of resource extraction under static, linear feedback and nonlinear feedback strategies, generalising his result that steady state feedback outputs are lower than monopoly and static oligopoly equilibrium outputs for any number of firms. Additionally, we show that (i) feedback rules entail resource exhaustion for a finite number of firms; and (ii) feedback strategies are more aggressive than static ones as long as the resource stock is large enough, in accordance with the acquired view based on the traditional pre-emption argument associated with feedback information.

JEL codes: C73, L13, Q2

Keywords: dynamic oligopoly, renewable resources, feedback strategies

1 Introduction

The analysis of dynamic market interplay through differential games has revealed - among other things - that feedback information boosts strategic interaction among firms as compared to open-loop information, triggering a pre-emption mechanism leading firms to expand production (for an overview, see Dockner *et al.*, 2000, ch. 10). Fujiwara (2008), relying on Benchekroun (2003, 2008), proposes a dynamic game of duopolistic extraction of a renewable resource, where at equilibrium output levels are lower under linear and nonlinear feedback information than under monopoly and the static oligopoly equilibria. We revisit his model allowing for the presence of n firms, to illustrate that his result that linear and nonlinear feedback equilibria are less competitive than monopoly and static oligopoly equilibria extend to the general case of an oligopoly with n firms.[1] This, however, implies that feedback information causes the exhaustion of the resource at the steady state for a finite number of firms (in correspondence of which equilibrium profits also drop to zero). This leads us to the main focus of our note, as the puzzling aspect of these results is that, taken together, they seem to imply that a less aggressive behavior goes along with exhaustion. The explanation lies in the fact that Fujiwara's appraisal is valid in steady state, but not at any generic instant during the game. Indeed, using the per-firm optimal output defined for a generic resource volume at a generic instant before doomsday, we show

[1] The extension to the case of n firms is mentioned in Fujiwara (2008, fn. 8, p. 219) while it is investigated in Benchekroun (2008) and Colombo and Labrecciosa (2013). The latter paper, in particular, focusses on the consequences of an *ex ante* resource parcelization among a population of firms. Fujiwara (2011) investigates the welfare effects of increasing the number of firms when these are characterised by different levels of technological efficiency.

that, as long as the amount of the resource is large enough, the traditional wisdom applies and output levels are larger under feedback rules, respecting the intuition behind the standard pre-emption argument. To illustrate this fact, we explicitly identify the critical threshold of the resource stock below which Fujiwara's conclusion holds true. Finally, we also illustrate the presence of a *voracity effect* operating for sufficiently high levels of the resource growth rate, whereby higher growth rates lead to lower steady state resource stocks.

2 The model

Consider a differential oligopoly game of resource extraction over time $t \in [0, \infty)$. The industry consists of an n firms producing a homogeneous good, whose inverse demand function is $p = a - Q$ at any time t, with $Q = \sum_{i=1}^{n} q_i$. Marginal cost $c \in (0, a)$ is constant and common to all firms, which operate without any fixed costs. During production, each firm exploits a renewable natural resource, whose accumulation is governed by the following dynamics:

$$\dot{S} = kS - Q \tag{1}$$

where S is the resource stock and $k > 0$ is its natural growth rate. If firms don't internalise the consequences of their behaviour at any time and play the individual (static) Cournot-Nash output $q^{CN} = (a - c) / (n + 1)$ at all times, whereby the residual amount of the natural resource in steady state is $S^{CN} = n (a - c) / [k (n + 1)] = Q^{CN}/k$. For future reference, it is worth noting that the static solution corresponds to the open-loop steady state one, which in this game is unstable (see Figure 1 in Fujiwara, 2008, p. 218; and Lambertini, 2013, p. 240).The initial condition is $S(0) = S_0 >$

$n(a-c)/[k(n+1)]$, which sufficies to guarantees $S>0$ at all times under the static Cournot-Nash strategies.[2]

3 Feedback Nash equilibria

Following Fujiwara (2008), we consider both linear feedback strategies *à la* Benchekroun (2003) and non linear strategies *à la* Tsutsui and Mino (1990) and Shimomura (1991). We restrict our attention to symmetric equilibria. The Hamilton-Jacobi-Bellman equation writes as:

$$rV_i(S)=\max_{q_i}\{[a-c-Q]q_i+V'(S)[kS-Q]q_i\} \tag{2}$$

where $r>0$ is the discount rate, common to all firms and constant over time; $V_i(S)$ is the firm i's value function; and $V'(S)=\partial V(S)/\partial S$. The first order condition (FOC) on q_i is

$$a-c-2q_i-\sum_{j\neq i}q_j-V'(S)=0 \tag{3}$$

In view of the *ex ante* symmetry across firms, we impose $q_j=q_i=q(S)$ and solve the FOC (3) to obtain $V'(S)=a-c-(n+1)q(S)$. Substituting this into (2) yields an identity in S. Differentiating both sides with respect to S and rearranging terms, any feedback strategy is implicitly given by the following differential equation:

$$q'(S)=\frac{(k-r)[(n+1)q(S)-(a-c)]}{2n^2q(S)-k(n+1)S-(n-1)(a-c)}, \tag{4}$$

[2] To see this, just observe that if firms always play *à la* Cournot, the stock at a generic t is

$$S(t)=\frac{n(a-c)+e^{kt}[k(n+1)S_0-n(a-c)]}{k(n+1)}$$

which is surely positive if the above condition holds.

which must hold together with terminal condition $\lim_{t\to\infty} e^{-rt}V(s)$. From Fujiwara (2008, p. 218), we borrow the assumption $k > 5r/2$, which amounts to requiring that the rate of reproduction of the natural resource be high enough to ensure the non negativity of steady state equilibrium magnitudes with $n \geq 2$. In general, to ensure non-negativity *for any number of firms*,one should assume $k > (n^2+1)\,r/2$, as in Benchekroun (2008, p. 240). The more restrictive assumption we are adopting is interesting for reasons that will become clear in the remainder.

3.1 Linear feedback strategy

If the strategy is linear in S, so that $q(S) = \alpha S + \beta$, equation (4) becomes:

$$\alpha = \frac{(k-r)\left[(n+1)(\alpha S+\beta)-(a-c)\right]}{2n^2(\alpha S+\beta)-(n+1)kS-(n-1)(a-c)} \tag{5}$$

which is satisfied iff

$$(k-r)\left[a-c-\beta\,(n+1)\right]+\alpha\left[2\beta n^2-(a-c)\,(n-1)\right]$$

$$+\alpha\left[r\,(n+1)-2\left(k\,(n+1)-\alpha n^2\right)\right]S=0. \tag{6}$$

The above equation gives rise to the following system of two equations

$$\begin{array}{c} \alpha\left[r\,(n+1)-2\left(k\,(n+1)-\alpha n^2\right)\right]=0 \\ (k-r)\left[a-c-\beta\,(n+1)\right]+\alpha\left[2\beta n^2-(a-c)\,(n-1)\right]=0 \end{array} \tag{7}$$

to be solved w.r.t. the unknown parameters $\{\alpha,\beta\}$. The pairs solving (7) are $(\alpha=0;\beta=(a-c)/(n+1))$, which replicates the static Cournot-Nash solution q^{CN}, and

$$\alpha=\frac{(n+1)\,(2k-r)}{2n^2};\ \beta=-\frac{(a-c)\left[2k-r\,(n^2+1)\right]}{2k\,(n+1)\,n^2}. \tag{8}$$

In correspondence of (8), the individual output is

$$q_{LF}^{N}(S)=\frac{k(2k-r)(n+1)^{2}S-(a-c)\left[2k-r\left(n^{2}+1\right)\right]}{2k(n+1)n^{2}} \tag{9}$$

where superscript N stands for *Nash equilibrium* while subscript LF stands for *linear feedback*. Leaving aside for brevity the replication of the stability analysis carried out by Fujiwara (2008, p. 218), we focus on (9). If $Q_{LF}^{*}=nq_{LF}^{*}$, the steady state amount of resource solving $\dot{S}=0$ is (henceforth, starred values indicate steady state equilibrium magnitudes):

$$S_{LF}^{*}=\frac{nq_{LF}^{*}}{k}=\frac{(a-c)\left[2k-r\left(n^{2}+1\right)\right]}{k\left[2k-r(n+1)\right](n+1)} \tag{10}$$

which is non-negative for all $k>r\left(n^{2}+1\right)/2$, the latter condition coinciding with the assumption $k>5r/2$ made by Fujiwara if $n=2$. It is then easily verified that

$$\frac{\partial Q_{LF}^{*}}{\partial n}=-\frac{2(a-c)(k-r)\left[2k+r\left(n^{2}-1\right)\right]}{(n+1)^{2}\left[2k-r(n+1)\right]^{2}}<0 \tag{11}$$

for all $n\geq 1$. However, it is also true that $S_{LF}^{*}=Q_{LF}^{*}=0$ for all $n\geq\sqrt{(2k-r)/r}>2$ (under the above assumption).

3.2 Nonlinear feedback strategy

The case of nonlinear feedback strategies can be quickly dealt with. One imposes stationarity on the state equation, obtaining $q=kS/n$, whereby (4) becomes:

$$\frac{k}{n}=\frac{(k-r)\left[k(n+1)S+n(a-c)\right]}{n(n-1)(a-c-kS)}, \tag{12}$$

from which one obtains

$$S_{NLF}^{*}=\frac{(a-c)(k-nr)}{k\left[2k-r(n+1)\right]}=\frac{nq_{NLF}^{*}}{k} \tag{13}$$

with $\partial S^*_{NLF}/\partial n \propto \partial Q^*_{NLF}/\partial n < 0$ for all $n \geq 1$, and $S^*_{NLF} = Q^*_{NLF} = 0$ for all $n \geq k/r > \sqrt{(2k-r)/r}$.[3]

The foregoing analysis can be summarised in

Lemma 1 *Under both linear and nonlinear feedback strategies, the steady state industry output is everywhere decreasing in the number of firms. However, so is also the steady state equilibrium resource stock, and both magnitudes drop to zero in correspondence of a finite number of firms, which is increasing in the resource growth rate and decreasing in the discount rate.*

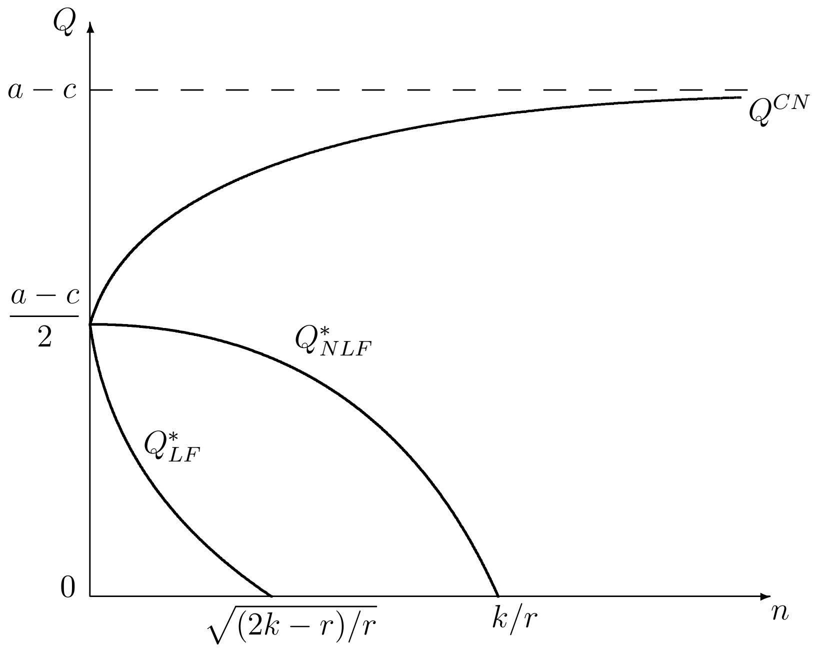
This generalises Fujiwara's conclusion to the general case of an oligopoly with n firms, making explicit an observation that can be found in Fujiwara (2008, fn. 8, p. 219) as to the fact that increasing the number of firms reduces aggregate extraction and output. However, may we really draw the implication that under feedback rules oligopolistic interaction is indeed less competitive than monopoly or static oligopoly? This question, which is a tricky one in connection with the exhaustion issue, is addressed in the next section.

3.3 Comparing equilibria

We are now in a position to comparatively assess firms' behaviour and its consequences across the three equilibria considered above. This exercise can be carried out graphically as in Figure 1, in the space $\{n, Q\}$, in which the curves representing the three possible aggregate outputs depart from the monopoly quantity $q_M = (a-c)/2$.

[3] The initial amount of resource must be lower than S^*_{NLF} in order for q^*_{NLF} to be an equilibrium strategy (see Itaya and Shimomura, 2001; Rubio and Casino, 202).

Figure 1 Equilibrium industry output and structure



Given the fixed proportion between S and Q, immediate implications can be drawn on the resource stock. Since the assumption $k > 5r/2$ is equivalent to $\sqrt{(2k-r)/r} > 2$, Figure 1 illustrates the following:

Proposition 2 *Feedback information leads to the exhaustion of the resource at the steady state for a finite number of firms, increasing in* k *and decreasing in* r*. Conversely, the residual resource stock at the static Cournot equilibrium is positive and increasing in* n*.*

It is worth noting that resource exhaustion, being accompanied by nil output levels, implies that steady state profits are also zero in correspondence of a finite number of firms, while the annihilation of profits at the static equilibrium takes place only in the limit as n tends to infinity and the industry becomes perfectly competitive.

From the existing literature (see Fershtman and Kamien, 1987; Reynolds, 1987, 1991; and Cellini and Lambertini, 2004, *inter alia*), we are accustomed to think that feedback information intensifies strategic interaction among firms, which translates into larger outputs due to the incentive to pre-empt rivals generated by feedback rules themselves. How can we reconcile this acquired wisdom with the seemingly opposite picture emerging from Proposition 2? That is, to what extent it is true that feedback strategies are less competitive than monopolistic behaviour and, *a fortiori*, static Cournot-Nash strategies?

To answer these questions, observe that the difference

$$q_{LF}^{N}(S) - q^{CN} = \frac{(2k-r)\left[k(n+1)^2 S - (a-c)(n^2+1)\right]}{2k(n+1)n^2} > 0 \tag{14}$$

for all

$$S > \frac{(a-c)(n^2+1)}{k(n+1)^2} \equiv \overline{S}, \tag{15}$$

with $\overline{S} > S_{LF}^{*}$ for all $k > (n+1)r/2$, which simplifies to $k > 5r/2$ if $n = 2$. This reveals that, as long as the resource stock is larger than the threshold $\overline{S}$, linear feedback strategies are indeed more aggressive than static Cournot ones. As soon as S drops below $\overline{S}$, the opposite applies throughout the continuation of the game, up to the steady state, where indeed the result portrayed in Proposition 2 and Figure 1 appears.

The last step consists in verifying whether, during the game, $Q_{LF}^{N}(S) = nq_{LF}^{N}(S) > q_M$ in an admissible range of S. It turns out that this holds true for all

$$S > \widehat{S} \equiv \frac{(a-c)\left[k(2+n(n+1)) - (n^2+1)r\right]}{k(2k-r)(n+1)^2} \tag{16}$$

with $\widehat{S} \in \left(S_{LF}^{*}, \overline{S}\right)$ for all admissible values of parameters. Hence, at any instant in which $S > \widehat{S}$, following linear feedback rules the oligopoly extracts and sells more than a monopolist. Thus, our analysis can be summarised in

Proposition 3 *Consider a generic instant* $t \in [0, \infty)$. *If, at time* t, $S > \overline{S}$, *then* $Q^N_{LF}(S) > nq^{CN}$ *for all* $n \geq 1$. *If instead* $S \in \left(\widehat{S}, \overline{S}\right)$, *then* $Q^N_{LF}(S) \in \left(q_M, nq^{CN}\right)$. *Finally, if* $S < \widehat{S}$, *then* $Q^N_{LF}(S) < q_M$.

Proposition 3 tells that the intensity of aggregate production (or resource extraction) at a generic point in time before the steady state is reached is decreasing in the existing stock of resource, falling below the monopoly level if the stock falls below a well defined threshold. Put it differently, the steady state picture does not encompass the behaviour of the industry while the game is still unraveling.

4 Voracity effect

Our exercise is also connected with the so-called *voracity effect* first explored in Lane and Tornell (1996) and Tornell and Lane (1999) and then investigated by Benchekroun (2008, pp. 245-48) using the same resource extraction game we have adopted here. In a nutshell, the voracity effect says that the *a priori* intuition suggesting that the higher is the resource growth rate, the higher should be the steady state volume of that resource, in fact may not be correct. This happens because a higher reproduction rate drives firms to hasten extraction, as indeed illustrated by (14-15) above. In this regard, we briefly complement the above analysis by looking at the comparative statics properties of the steady state levels of S in the three cases under examination:

$$\begin{gathered}\frac{\partial S^{CN}}{\partial k} = -\frac{n(a-c)}{(n+1)k^2} < 0 \text{ everywhere} \\ \frac{\partial S^*_{LF}}{\partial k} = -\frac{(a-c)\left[(n+1)(n^2+1)r^2 + 4k\left(k-(n^2+1)r\right)\right]}{(n+1)\left[2k - r(n+1)\right]^2 k^2} < 0 \forall k > \widetilde{k} \\ \frac{\partial S^*_{NLF}}{\partial k} = -\frac{(a-c)\left[n(n+1)r^2 + 2k(k-2nr)\right]}{\left[2k-(n+1)r\right]^2 k^2} < 0 \forall k > \widehat{k}\end{gathered} \tag{17}$$

with

$$\widetilde{k} = \frac{r}{2}\left[n^2 + 1 + \sqrt{(n^2+1)(2+n(n+1))}\right]$$
$$\widehat{k} = r\left[n + \sqrt{\frac{n(3n+1)}{2}}\right] \tag{18}$$

and $\widetilde{k} > \max\left\{\widehat{k}, (n^2+1)\, r/2\right\}$. It is also easily ascertained that $\widetilde{k}$ and $\widehat{k}$ are increasing and convex in n. This allows us to formulate our final result:

Proposition 4 *$k > \widetilde{k}$ suffices to ensure that the steady state resource stock be decreasing in the growth rate, irrespective of the structure of information underlying firms' equilibrium strategies. Under feedback rules, increasing the number of firms makes the appearance of voracity progressively less likely.*

5 Concluding remarks

Revisiting the dynamic game of renewable resource extraction by Benchekroun (2008) and Fujiwara (2009), we have singled out a feature that has been previously overlooked, namely, that feedback strategies, although appearing less aggressive than static ones in steady state, indeed imply a higher pressure on the resource on the part of firms, whereby the steady state stock may indeed be driven to zero at equilibrium for a finite number of firms. This can be explained on the basis of a pre-emtpion incentive operating during the game, accompanied by a voracity effect if the growth rate of the resource is high enough.

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Strada Maggiore 45
40125 Bologna - Italy
Tel. +39 051 2092604
Fax +39 051 2092664
http://www.dse.unibo.it